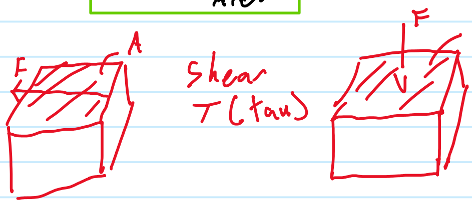
Stress

Friday, January 20, 2023 8:58 AM

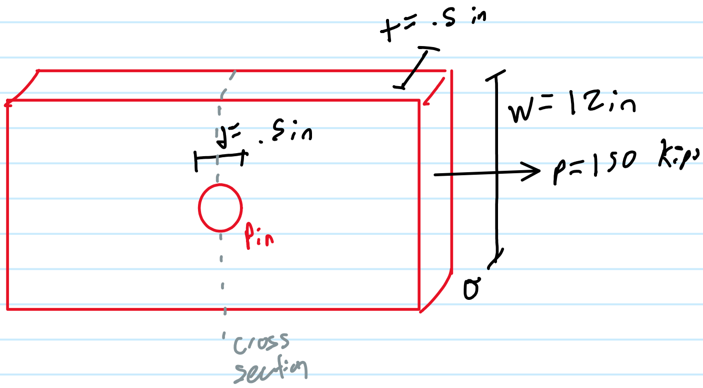
Internal force: stress σ(sigma), τ(tau)

$$\boxed{\sigma = \frac{\text{Force}}{\text{Area}}}$$



Bearing stress is a subset of normal stress

Ex.



Normal:

$$\sigma = \frac{F}{A} = \frac{P}{(w-d)t} = \frac{150}{(12-.5)} \cdot 1000 = 20\,6090 \text{ psi}$$

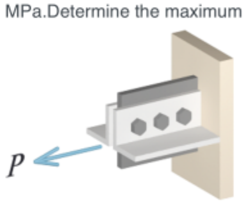
Bearing:

$$\sigma = \frac{F}{A} = \frac{P}{td} = \frac{150}{(.5)(.5)} \cdot 1000 = 600{,}000 \text{ psi}$$

Shear:

$$\tau = \frac{F}{A} = \frac{P}{\pi r^2} = \frac{P}{\frac{\pi}{4}d^2} = \frac{150}{\frac{\pi}{4}(.5)^2} \cdot 1000 = 763943 \text{ psi}$$

For the connection shown, the average shear stress in the 16 mm-diameter bolts must be limited to τ_{all} = 230 MPa.Determine the maximum load P that may be applied to the connection. [τ_{all}= 230 MPa]



$$\tau = \frac{P}{A} = \frac{P}{2\frac{\pi}{4}d^2 \cdot n}$$

Area is doubled since there are two plates

$$230 \times 10^3 = \frac{P}{2\frac{\pi}{4}(16)^2 \cdot 3} \quad N$$

Pa

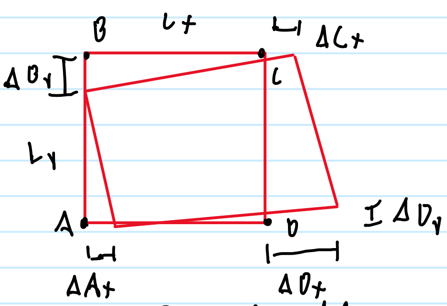
- [ ] 378.7kN
- [ ] 311kN
- [x] 277.5kN
- [ ] 99.1kN
- [ ] 424.7kN

Normal Strain

Monday, January 23, 2023 8:58 AM

Strain: intensity of internal deformation

$$\boxed{\text{Normal Strain} = \frac{\text{change in length } (\delta)}{\text{initial length } (L)} \quad [\varepsilon]}$$



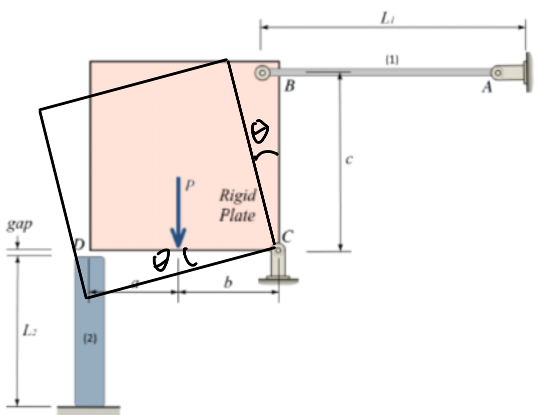
Find at A

$$\delta_x = \Delta D_x - \Delta A_x = .6 - .4 = .2$$

$$\varepsilon_x \; \frac{\delta_x}{L_x} = \frac{.2}{450} = 444\ u\varepsilon$$

$$\varepsilon_y = \frac{-.35}{300} = -1166\ u\varepsilon$$

The load P produces an absolute axial strain of 1150 μm/m in post (2).
[L_1= 900 mm, L_2 = 1000 mm, a=b= 300 mm, c= 420 mm, gap= 1.6 mm]



Determine the axial strain in rod (1).

$$\varepsilon_2 = 1150\ u\varepsilon$$

$$\varepsilon_2 = \frac{\delta_2}{L_2} \cdot 10^6 \qquad\qquad \frac{\delta_2 + gap}{a+b} = \frac{\delta_1}{c}$$

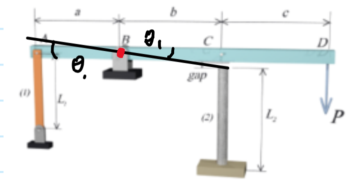
$$\varepsilon_1 = \frac{\delta_1}{L_1} \cdot 10^6 = \frac{c(\delta_2 + gap)}{(a+b)L_1} = \frac{c(L_2\varepsilon_2/10^6 + gap)\,10^6}{(a+b)L_1}$$

$$\varepsilon_1 = 420(1000 \cdot 1150/10^6 + 1.6)\,10^6 = 2138.9\ u\varepsilon$$

$$\frac{\ }{L_1} \qquad (a+b)L_1 \qquad (a+b)L_1$$

$$\varepsilon_1 = \frac{420\left(1000 \cdot 1150/10^6 + 1.6\right)}{(300+300)\,900} \cdot 10^6 = 2138.9\ u\varepsilon$$

A load P is applied at the end of a rigid beam that is just sufficient to close the gap between the rigid beam and the top of the column (2) at C. Then, the load P is increased until $\varepsilon_2 = 1220\ \mu\varepsilon$.[a = 4 ft,b = 6 ft, c = 4 ft, L_1 = 5 ft, L_2 = 3 ft, gap = 0.25 in.]



What is the strain in element (1) for the original load P, just sufficient enough to close the gap at C?

$$\varepsilon_1 = \frac{\delta_1}{L_1} \cdot 10^6 \qquad \frac{\delta_1}{a} = \frac{gap}{b}$$

$$\varepsilon_1 = \frac{gap \cdot a}{b \cdot L_1} \cdot 10^6 = \frac{.25/12 \cdot 4 \cdot 10^6}{6 \cdot 5} = 2777\ u\varepsilon$$

$$\varepsilon_2 = \frac{\delta_2}{L_2} \cdot 10^6 \qquad \delta_2 = \frac{\varepsilon_2 L_2}{10^6}$$

$$\varepsilon_1 = \frac{\delta_1}{L_1} \cdot 10^6 \qquad \frac{\delta_1}{a} = \frac{\delta_2 + gap}{b}$$

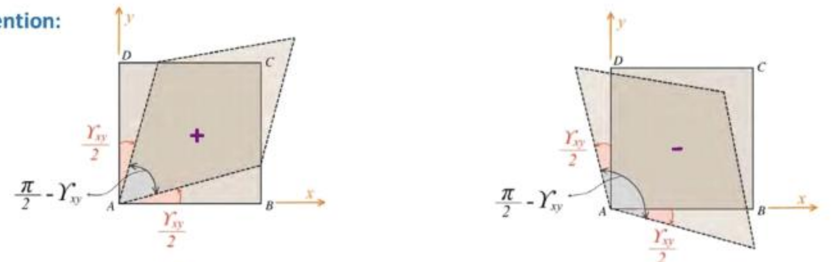
$$\varepsilon_1 = \frac{a(\delta_2 + gap)}{b\ L1}\ 10\text{^}6$$

Shear Strain

Tuesday, January 24, 2023 3:05 PM

Intensity of change in the angle of the element. Deformation is parallel to the face of the element.

Sign convention:

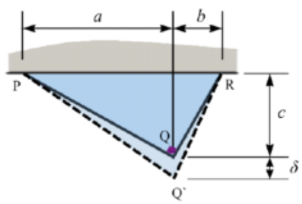


A positive value for the shear strain means that the angle between the x and y axes decreases in the deformed object.

The angle between the x and y axes increases when the shear strain has a negative value

---

A thin polymer plate PQR is deformed such that corner Q is displaced downward δ = 0.5 in. to new position Q' as shown in the figure.[a = 31.25 in., b = 20 in., c = 25 in.]



Determine the shear strain at Q associated with the two edges (PQ and QR).

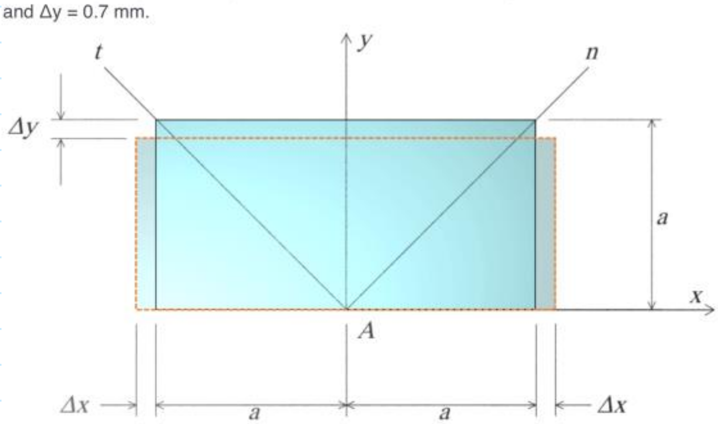
$\tan \alpha_1 = \frac{b}{c+\delta}$

$\tan \alpha_2 = \frac{a}{c+\delta}$

$\gamma = \frac{\pi}{2} - (\alpha_1 + \alpha_2) = \frac{\pi}{2} - \left(\tan^{-1}\left(\frac{b}{c+\delta}\right) + \tan^{-1}\left(\frac{a}{c+\delta}\right)\right)$

$\gamma = \frac{\pi}{2} - \left(\tan^{-1}\left(\frac{20}{25+.5}\right) + \tan^{-1}\left(\frac{31.25}{25+.5}\right)\right) = 19318\ \mu rad$

The rectangular plate is subjected to the deformation shown by the dashed line. Assume a = 550 mm, Δx = 2.4 mm, and Δy = 0.7 mm.



Determine the shear strain γ_{xy} at point A.

0

Find γ_{xy} at A

$$\gamma_{xy} = \left(\tan^{-1}\left(\frac{a}{a}\right) - \tan^{-1}\left(\frac{a-\Delta y}{a+\Delta x}\right)\right)\cdot 10^6$$

Find normal strain.

$$\varepsilon_x = \frac{\Delta x}{a}\cdot 10^6$$

$$\varepsilon_y = \frac{-\Delta y}{a}\cdot 10^6$$

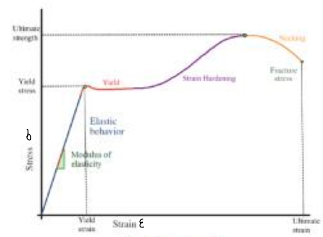
$$\varepsilon_n = \frac{\sqrt{(a^2+a^2)} - \sqrt{(a-\Delta y)^2+(a+\Delta x)^2}}{\sqrt{a^2+a^2}}\cdot 10^6$$

Mechanical Properties

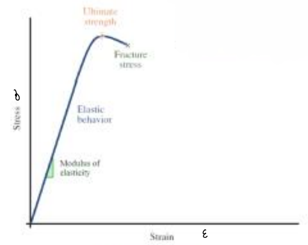
Friday, January 27, 2023 8:59 AM

Young's Modulus (Modulus of elasticity) (E):

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$



Ductile material



Brittle material

Poisson's ratio (v):

$$\boxed{v = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}}$$

$$0 \le v < 0.5$$

Material	Poisson's ratio
aluminum-alloy	0.32
cast iron	0.21–0.26
concrete	0.1-0.2
copper	0.33
glass	0.18–0.3
rubber	0.4999
steel	0.27–0.30

Modulus of rigidity (G):

relates shear strain to shear stress

$$\boxed{G = \frac{E}{2(1+v)}}$$

Thermal strain (ε_T):

$$\boxed{\varepsilon_T = \alpha \Delta T}$$

Thermal deformation (δ_T):

$$\boxed{\delta_T = \varepsilon_T L = \alpha \Delta T L}$$

Coefficient of Thermal Expansion (CTE) (α):

Degree of expansion divided by change in temp

$$\boxed{\text{Total strain} = \varepsilon_e + \varepsilon_T}$$

Elastic strain: $\varepsilon_e = \frac{\sigma}{E}$

CTE values for different materials

Material	CTE (/°C)
aluminum-alloy	23.1×10^{-6}
cast iron	11.8×10^{-6}
concrete	8×10^{-6} - 12×10^{-6}
copper	17×10^{-6}
glass	8.5×10^{-6}
Quartz	0.33×10^{-6}
steel	11.5×10^{-6}

An alloy bar with an initial length of 10 in. and diameter of 0.76 in. is subjected to a tensile force of 13.75 kips. Due to the force, the bar has elongated 0.02 inches, and the change in the diameter is measured as 0.0005 inches.

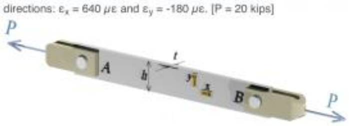
Determine the modulus of elasticity.

$$\sigma = \frac{F}{\frac{\pi}{4} d^2} \qquad E = \frac{\sigma}{\varepsilon}$$

$$\varepsilon_{long} = \frac{\delta}{L}$$

$$\varepsilon_{lat} = \frac{\Delta d}{d}$$

A 0.5 -in. thick rectangular alloy bar is subjected to a tensile load P by pins at A and B. The width of the bar is h = 3.5 in. Strain gages bonded to the specimen measure the following strains in the longitudinal (x) and transverse (y) directions: ε_x = 640 $\mu\varepsilon$ and ε_y = -180 $\mu\varepsilon$. [P = 20 kips]

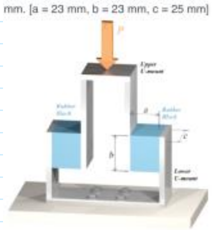


Determine Poisson's ratio for this specimen.

$$v = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$E = \frac{\sigma}{\varepsilon_x} \qquad \sigma = \frac{F}{A} = \frac{P}{wh}$$

The 23 × 23 × 25 -mm rubber blocks shown in the figure are used in a double U shear mount to isolate the vibration of a machine from its supports. An applied load of P = 415 N causes the upper frame to be deflected downward by 4.5 mm. [a = 23 mm, b = 23 mm, c = 25 mm]



Determine the average shear stress in the rubber blocks.

$$\tau = \frac{F}{A} = \frac{P/2}{b \cdot c}$$

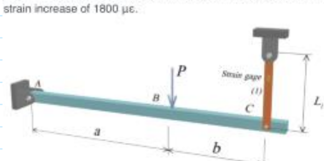
Shear strain:

$$\tan \gamma = \frac{\delta}{a}$$

G:

$$\tau = G\gamma \qquad G = \frac{\tau}{\gamma}$$

Rigid bar ABC is supported by member (1), which has a cross-sectional area of 2 in.^{2}, an elastic modulus of E = 10900 ksi, and a coefficient of thermal expansion of α = 12.6 *10^{-6}/°F. Assume that a = 50 in., b = 25 in., and L_1 = 60 in. After load P is applied to the rigid bar and the temperature rises 65 °F, a strain gage affixed to member (1) measures a strain increase of 1800 $\mu\varepsilon$.



Determine the magnitude of applied load P.

1. $\Sigma M_A = -aP + (a+b)F = 0$

$$F = \frac{aP}{a+b}$$

2. $\sigma = \frac{F}{A}$

3. $E = 10900 = \frac{\sigma}{\varepsilon_e}$

4. $\varepsilon = \varepsilon_e + \varepsilon_T$

4. $\varepsilon = \varepsilon_e + \varepsilon_T$

$\varepsilon_e = \varepsilon - \varepsilon_T$

5. $\varepsilon_T = \alpha \Delta T$

$\Delta T = 65°F$

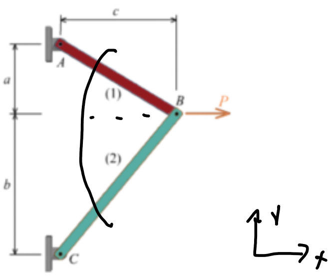
$$P = \frac{(a+b)F}{a} = \frac{(a+b)\,\sigma A}{a} = \frac{(a+b)AE\varepsilon_e}{a} = \frac{(a+b)AE(\varepsilon - \varepsilon_T)}{a}$$

$$P = \frac{(a+b)AE(\varepsilon - \alpha \Delta T)}{a}$$

Design Examples

Monday, January 30, 2023 3:32 PM

A concentrated load P is supported by two inclined bars as shown in the figure. Bar (1) is made of cold-rolled stainless steel [σ_y = 170 ksi] and has a cross sectional area of 2.05 in.^{2}. Bar (2) is made of 6061-T6 aluminum [σ_y = 40 ksi] and has a cross sectional area of 9 in.^{2}. A factor of safety of 1.67 is required for both bars. Dimensions of the assembly are a = 9 ft, b = 23 ft, and c = 12 ft.



Determine the magnitude of load P that can be applied to this assembly considering the stress in bar (1)

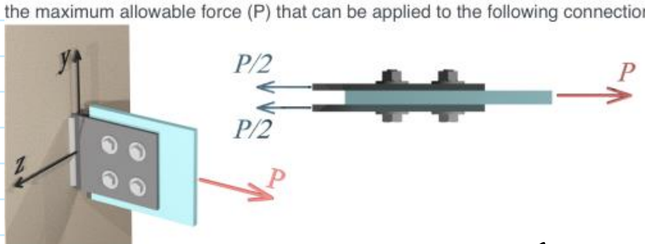
$$\Sigma F_x = P - F_1 \frac{c}{\sqrt{a^2+c^2}} - F_2 \frac{c}{\sqrt{b^2+c^2}} = 0$$

$$\Sigma F_y = F_1 \frac{a}{\sqrt{a^2+c^2}} - F_2 \frac{b}{\sqrt{b^2+c^2}} = 0$$

$$\frac{\sigma_{y1}}{1.67} = \frac{F_1}{A_1} = 232 \text{ kips}$$ lower, so maximum allowable

$$\frac{\sigma_{y2}}{1.67} = \frac{F_2}{A_2} = 355 \text{ kips}$$

An axial element that is subjected to force P is connected by two plates and 4 bolts as shown in the figure. The ultimate shear stress of the pins is τ_y = 21 ksi. The pins diameter is d = 0.75 in. The factor of safety is 1.59 .Determine the maximum allowable force (P) that can be applied to the following connection.

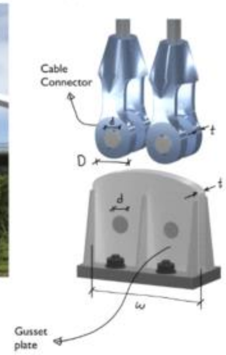


$$\frac{\tau}{1.59} = \frac{F}{4 \cdot 2 \cdot \frac{\pi}{4} \cdot d^2}$$

- 95.4 kips
- 72.9 kips
- 46.7 kips
- 168.2 kips
- 145.7 kips

The bridge shown in the figure features a unique inclined single rib arch that holds the deck with cables and struts. The cable connection of a cable-stayed bridge is shown in the figure. Determine the maximum allowable force that can be carried by the connection by assuming the following parameters: Gusset plate: [w_g= 18 in. , t_g= 2.2 in., σ_{yg}= 37 ksi, FS_g = 1.79].Cable connector: [D = 5 in., t_c= 1.2 in., σ_{yc}= 61 ksi, FS_c= 2.4].Bolts: [d= 2.5 in., τ_y= 27 ksi, FS_b= 3.1].


Main street bridge, Columbus, Ohio



What is the critical cross section area in the gusset plate for the calculation of the normal stress in the plate?

$$\frac{\sigma_{yg}}{FS_g} = \frac{F_g}{(w_g - 2d)\,t_g}$$

$$\frac{\sigma_{yg}}{FS_g} = \frac{F_g}{2\,d\,t_g}$$

$$\frac{\sigma_{yc}}{FS_c} : \frac{F_c}{2\,D\,t_c}$$

Same ?

$$\frac{\tau_{yb}}{FS_b} = \frac{F_b}{\frac{\pi}{4} D^2}$$

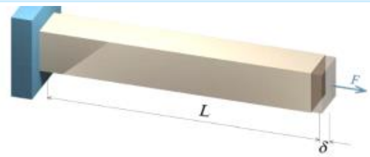
Deformation in Axial Members

Wednesday, February 1, 2023 8:53 AM

Simple axial element

One element, constant force (F), constant cross-sectional area (A), made from one material (constant E)

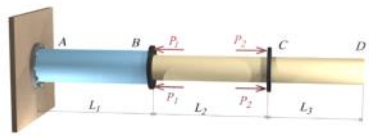
$$\delta = \frac{FL}{EA}$$



System of connected axial elements

Each segment has a constant force and a constant cross-sectional area

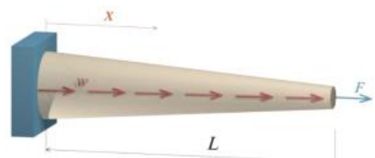
$$\delta = \sum \frac{F_i L_i}{E_i A_i}$$



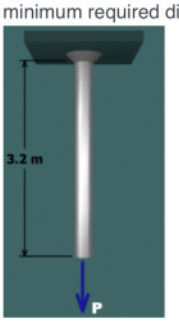
Axial element with variable section/loading

where the force or cross-sectio (A) are not constant

$$\delta = \int_0^L \frac{F(x)}{E(x)A(x)} dx$$



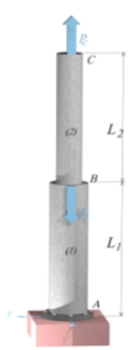
A steel [E = 130 GPa] rod with a length of L = 3.2 m is carrying an axial tensile force of P = 90 kN. What is the minimum required diameter of the rod if the axial elongation of the rod is limited to δ_{max} = 17 mm?



$$\delta_{Max} = \frac{P \cdot L}{E \frac{\pi}{4} d^2}$$

- 18.59 mm
- 29.14 mm
- 14.98 mm
- 12.88 mm
- 25.7 mm

The system shown in the figure consists of two bars and is subjected to a force at B and C.[d_1 = 53 mm, d_2 = 33 mm, L_1 = 1.3 m, L_2 = 1.3 m, E= 120 GPa, P_C = 29 kN, P_B = 39 kN]



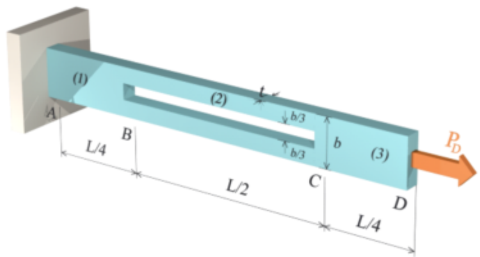
$$F_1 = P_B - P_C$$

$$\sigma = \frac{F_1}{\frac{\pi}{4} d_1^2}$$

Determine the normal stress in rod (1).

$$\delta = \frac{F_1 L_1}{E \frac{\pi}{4} d_1^2}$$

A rectangular bar of length L = 650 mm has a slot in the central half of its length. The bar has a width of 42 (b = 42 mm) and thickness of 5 mm (t = 5 mm). The slot has a width of b/3. The elastic modulus of the material is E = 79 GPa. The average normal stress in the central portion of the bar is σ = 190 MPa.



$$\delta_1 = \frac{FL}{EA} = \frac{\sigma\, t\, \frac{2}{3} b\, L/4}{E\, t \cdot b}$$

$$\sigma = \frac{F}{A}$$

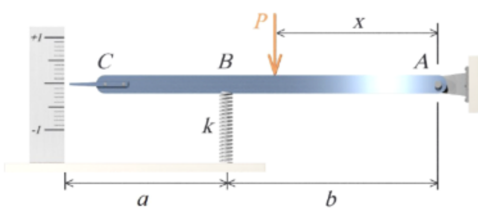
$$P_D = \sigma A$$

$$P_D = \sigma\, t\, \frac{2}{3} b$$

$$\delta_2 = \frac{FL}{EA} = \frac{\sigma A\, L}{EA} = \frac{\sigma\, L/2}{E}$$

Determine the elongation in section (1).

A simple balance is made by a horizontal handle, a spring with a known spring constant, k= 340 N/mm and a ruler as shown in the figure. After the force P is applied on on the handle, the downward movement of the pointer on the left side of handle is measured by the ruler as δ_C = 1.2 mm. [b= 250 mm, a = 140 mm, x = 190 mm, k= 340 N/mm]



What is the absolute value of deformation in the spring (at B)?

$$F = k\, n \qquad \frac{\delta_C}{a+b} = \frac{n}{b}$$

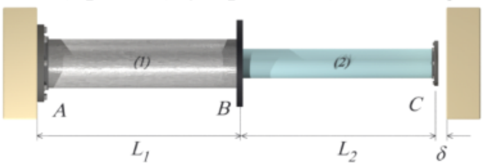
$$\frac{\cdot}{x} = \frac{F}{b}$$

$$\delta = \int_0^L \frac{F}{EA}\, dx$$

Indeterminate Axial Members

Friday, February 3, 2023 1:22 PM

A pipe-rod system with flanges at ends A and C was supposed to fit exactly between two rigid walls, as shown in the figure. Element (1) is a steel pipe and element (2) is a solid steel rod. Bolts hold the flange at A against a rigid wall. Other bolts are installed in the flange at C and are tightened until the gap is closed. [A_1 = 321 mm^2, A_2 = 296 mm^2, L_1 = 61 cm, L_2 = 41 cm, $E_1 = E_2$ = 200 GPa, δ = 0.25 mm.]



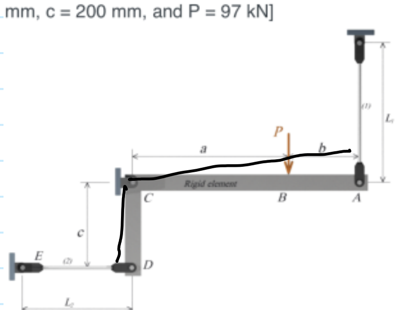
Determine the internal force in element (1) when the gap at C has been closed.

$$F_1 = F_2$$

$$\delta_1 = \frac{F_1 L_1}{E A_1} \qquad \delta_2 = \frac{F_2 L_2}{E A_2}$$

$$\delta_1 + \delta_2 = \delta$$

The rigid element ABCD is supported by a pin at C and two rods at A and D as shown in the figure. A load P is applied at B.[L_1 = 550 mm, A_1 = 65 mm^2, E_1 = 200 GPa, L_2 = 170 mm, A_2 = 628 mm^2, E_2 = 100 GPa, a = 550 mm, b = 250 mm, c = 200 mm, and P = 97 kN]



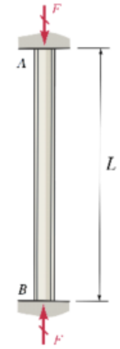
Determine the axial force in rod (1).

$$\Sigma M_C = -P \cdot a + F_1(a+b) - F_2 \cdot c \qquad -\frac{\delta_1}{a+b} = \frac{\delta_2}{a}$$

$$\delta_1 = \frac{F_1 L_1}{E_1 A_1}$$

$$\delta_2 = \frac{F_2 L_2}{E_2 A_2}$$

A prismatic bar AB of length 375 mm is held between immovable supports. The temperature of the bar is raised uniformly by an amount 39. Assume the bar is made of steel [E= 200 GPa; α= 11.7 x10^{-6}/C].



$$\varepsilon_T = \alpha \Delta T$$

$$E = \frac{\sigma}{\varepsilon}$$

$$\sigma = \varepsilon E = E \alpha \Delta T$$

What is the thermal stress developed in the bar?

- -137.4 MPa
- -91.3 MPa
- -181.3 MPa
- -150 MPa
- -108.8 MPa

A reinforced concrete column with four steel bars (d = 1 in.) is shown in the figure. The column is subjected to an axial force of P = 189 kips.[E_s = 29000 ksi, E_c = 3600 ksi, b = 13 in., and h = 9 ft.]

$$F_s + F_c = -P$$

$$\delta_s = \frac{FL}{EA} = \frac{F_s h}{E_s \frac{\pi}{4} d^2 4}$$

Determine the absolute value of the axial force in the steel bars.

$$\delta_c = \frac{F_c h}{E_c \left(b^2 - \frac{\pi}{4} d^2 4\right)}$$

$$\delta_s = \delta_c$$

« Previous 1 2 3 4 Next » 20

An aluminum tube with a cross-sectional area of A_s = 655 mm^2 is used as a sleeve for a steel bolt having a cross-sectional area of A_b = 325 mm^2. When the temperature is T_1= 15 °C, the nut holds the assembly in a snug position such that the axial force in the bolt is negligible. Assume that the temperature increases to T_2= 71 °C. [Aluminum sleeve: E_s= 73 GPa, α_s = 23 ×10^{-6} /°C, A_s = 655 mm^2; Steel bolt: E_b = 200 GPa, α_b = 12 ×10^{-6} /°C, A_b = 325 mm^2]

$$F_s + F_b = 0$$

$$\delta_s = \frac{F_s L}{E_s A_s}$$

$$\delta_b = \frac{F_b L}{E_b A_b}$$

What is the elastic deformation for the sleeve in terms of the internal force?

$$\delta = \delta_s + \alpha_s (T_2 - T_1) L + \delta_b + \alpha_b (T_2 - T_1) L$$

physics 10 26

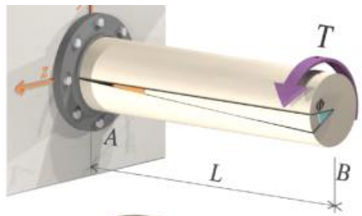
Torsion

Friday, February 10, 2023 9:21 AM

Shear Stress

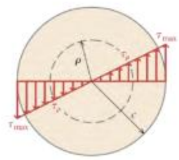
$\tau_{max} = \frac{Tc}{J}$ $\tau = \frac{T\rho}{J}$

T: internal torque in the shaft
c or ρ: distance of the point at which the stress is being calculated from the centroid of section
J: polar moment of inertia



Internal torque (T) is determined using free body diagram:

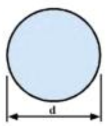
- Cut the element at section where torque should be determined
- Put internal torque (T) at the cut section outward from the surface
- Determine T using equilibrium equation of torques



Polar moment of inertia

Solid Shaft

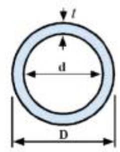
$J = \frac{\pi}{2}r^4 = \frac{\pi}{32}d^4$



Tubular Shaft

$J = \frac{\pi}{2}[R^4 - r^4] = \frac{\pi}{32}[D^4 - d^4]$

$d = D\text{-}2t$



A steel shaft with an outside diameter of d = 100 mm is subjected to a pure torque of T = 2360 N-m. The shear modulus of the steel is G = 79 GPa. Determine the maximum shear stress in the shaft.

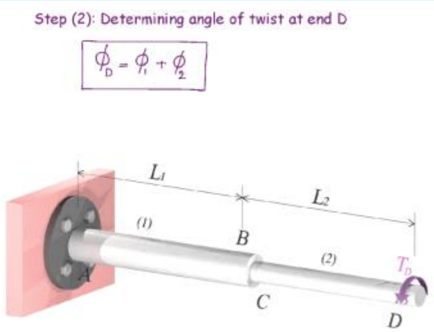
- ○ 16.9 MPa
- ○ 14.7 MPa
- ● 12 MPa
- ○ 10.4 MPa
- ○ 8.2 MPa

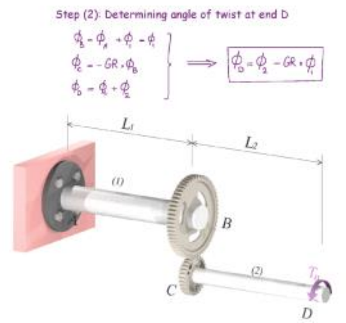
$$\tau = \frac{Tc}{J} = \frac{T\frac{d}{2}}{\frac{\pi}{32}d^4}$$

Twist

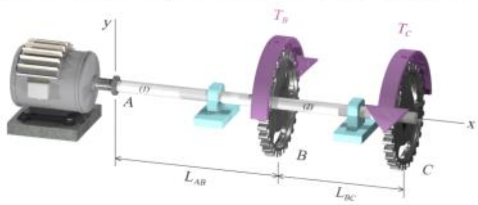
Wednesday, February 15, 2023 9:06 AM

$$\phi = \frac{TL}{GJ}$$





The solid steel (G = 80 GPa) shaft between A and B has a diameter of 34 mm and between B and C has a diameter of 27 mm. A 20 N-m concentrated torque is applied at gear B and a concentrated torque T_C is applied at gear C as shown. The total angle of rotation at C should be equal to Φ_C = 1.8 °. [L_1 = 1400 mm, L_2 = 1000 mm]



Determine the magnitude of torque T_C.

$$\phi_C = \phi_1 + \phi_2$$

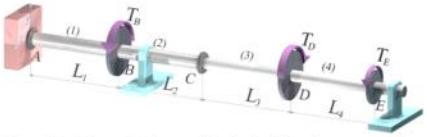
$$\phi_C = \frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} \qquad T_C = T_2$$

$$T_1 = T_C - T_B$$

$$\phi_C = \frac{(T_C - T_B)\ L_1}{G \frac{\pi}{32} d_1^4} + \frac{T_C L_2}{G \frac{\pi}{32} d^4}$$

$$1.8\left(\frac{\pi}{180}\right) = \frac{(T_C - 27 \cdot 10^3)1400}{80 \cdot 10^3 \frac{\pi}{32}(34)^4} + \frac{T_C(1000)}{80 \times 10^3 \frac{\pi}{32}(27)^4}$$

A compound shaft drives several pulleys, as shown in the figure. Segments (1) and (2) of the compound shaft are hollow aluminum [G_{12} = 4000 ksi] tubes that have a polar moment of inertia of J_{12} = 2.328 in.^{4}. Segments (3) and (4) are solid steel [G_{34} = 13000 ksi] shafts that have a polar moment of inertia of J_{34} = 0.508 in.^{4}. The bearings shown allow the shaft to turn freely. [L_1 = 75 in., L_2 = 30 in., L_3 = 35 in., L_4 = 20 in., T_B = 925 lb-ft, T_D = 525 lb-ft, T_E = 170 lb-ft].



Calculate the rotation angle (including the correct sign) of pulley D with respect to pulley B.

- -0.0289 rad
- -0.0363 rad
- -0.01062 rad
- -0.02474 rad
- -0.03226 rad

$\frac{lb}{ft}$ $\frac{1ft}{12in}$

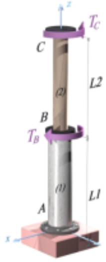
$\phi = \frac{TL}{GJ}$

$T_2 = T_E - T_D$

$T_3 = T_E - T_D$

$$\phi_2 = \frac{(T_E - T_D)30}{4000(2.328)} + \frac{(T_E - T_D)35}{13000(.508)}$$

The compound shaft shown in the figure consists of a d_1 = 37 mm solid bronze [G_{bronze} = 40 GPa] shaft (1) and a 29 mm solid steel [G_{steel} = 85 GPa] shaft (2). The compound shaft is subjected to torques of 830 N.m and 350 N.m at B and C, respectively. Let L_1 = 750 mm and L_2 = 1280 mm.

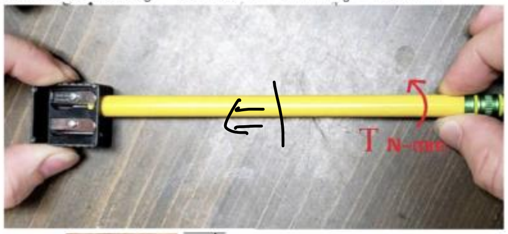


Determine the internal torque in shaft (2).

Assume you are attempting to sharpen a pencil in a pencil sharpener when it becomes jammed causing it to act as fixed support. A torque of T= 80 N-mm is required to free the jam. Assume the graphite and wood are fully connected together. Pencil length L = 100 mm,

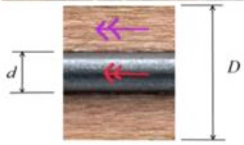
Pencil wood: G_w = 14 GPa, D = 9 mm, J_w = 636.2 mm^4

Pencil graphite: G_g = 40 GPa, d = 3 mm, J_g = 7.95 mm^4



1. $T_g + T_w + T = 0$

2.



What is the absolute value of torque in the the graphite inside the pencil rightbefore the pen is free.

$$\phi_g = \frac{TL}{GJ} = \frac{T_g \, 100}{40 \times 10^7} \quad 7.95$$

$$\phi_w = \frac{T_w \, 100}{14 \times 10^3} \quad 636.2$$

$$-T_1 + T_2 + T_B = 0$$

$$\phi_1 + \phi_2 = 0$$

$$\phi_1 = \frac{T_1 \, 300}{86 \frac{\pi}{32} 70^4}$$

$$\phi_2 = \frac{T_2 \, 400}{45 \frac{\pi}{32} 5^4}$$

$$T_{1\,all} = 84 = \frac{T_1 \frac{d_1}{2}}{\frac{\pi}{32}(d_1^4 - d_2^4)}$$

$$T_{2\,all} = 67 = \frac{T_2 \frac{d_2}{2}}{\frac{\pi}{32}(d_2^4 - d_3^4)}$$

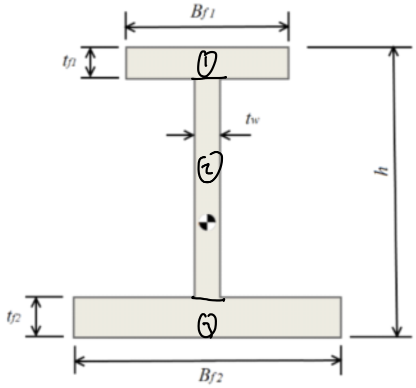
$$-T_1 - T_2 + T = 0$$

$$T = T_1 + T_2$$

Centroid and Moment of Inertia

Monday, February 20, 2023 1:55 PM

Determine the following section properties for the section shown in the figure.
[B_{f1} = 70 mm, t_{f1} = 14 mm, h = 310 mm, B_{f2} = 110 mm, t_{f2} = 19 mm, t_w = 9 mm]



Determine the distance of centroid from the bottom of the section.

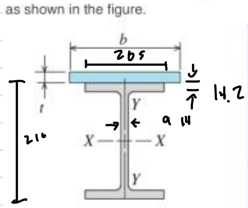
$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{709447.5}{5563} = 127.5\text{ mm}$$

	A_i	y_i	$A_i y_i$
1	14(70)	310 − 14/2	296940
2	9(310 − 19 − 14)	$\frac{(310-19-14)}{2}$ + 19	392647.5
3	19(110)	19/2	19855
Σ	5563		709442.5

$$I_x = \Sigma(I_{cx} + A_i d_i^2) = 16{,}007 + 30{,}184{,}245 + 15{,}940{,}450 + 2{,}243{,}700 + 62{,}874 + 29{,}501{,}160$$
$$= 77.5484 \times 10^6\text{ mm}$$

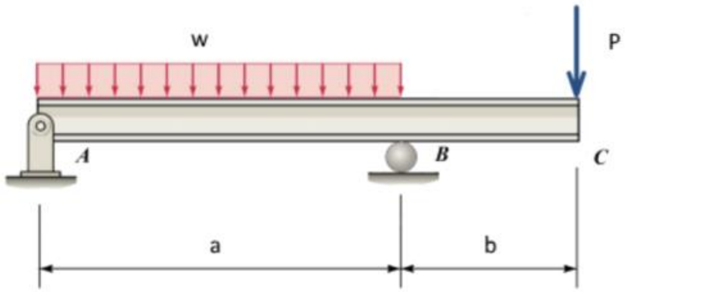
	A_i	d_i	$A_i d_i^2$	$I_c = \frac{bh^3}{12}$
1	980	127.5 − 303	30,184,245	$\frac{70(14)^3}{12}$ = 16,007
2	2493	127.5 − 157.5	2,243,700	$\frac{9(277)^3}{12}$ = 15,940,450
3	2090	127.5 − 9.5	29,501,160	$\frac{110(19)^3}{12}$ = 62,874

A W200×59 standard steel shape is strengthened by adding a b= 245 mm × t= 11 mm plate on the top of the section as shown in the figure.



Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	I_x	S_x	r_x	I_y	S_y	r_y
	mm²	mm	mm	mm	mm	10^6 mm⁴	10^3 mm³	mm	10^6 mm⁴	10^3 mm³	mm
W250 × 80	10200	257	9.40	254	15.6	126	983	111	42.9	338	65.0
250 × 67	8580	257	8.89	204	15.7	103	805	110	22.2	218	51.1
250 × 44.8	5700	267	7.62	148	13.0	70.8	531	111	6.95	94.2	34.8
250 × 38.5	4910	262	6.60	147	11.2	59.9	457	110	5.87	80.1	34.5
250 × 32.7	4190	259	6.10	146	9.14	49.1	380	108	4.75	65.1	33.8
250 × 22.3	2850	254	5.84	102	6.86	28.7	226	100	1.20	23.8	20.6
W200 × 71	9100	216	10.2	206	17.4	76.6	708	91.7	25.3	246	52.8
200 × 59	7550	210	9.14	205	14.2	60.8	582	89.7	20.4	200	51.8
200 × 46.1	5880	203	7.24	203	11.0	45.8	451	88.1	15.4	152	51.3
200 × 35.9	4570	201	6.22	165	10.2	34.4	342	86.9	7.62	92.3	40.9
200 × 22.5	2860	206	6.22	102	8.00	20	193	83.6	1.42	27.9	22.3

Given w = 495 lb/ft, P = 1200 lbs, a = 14 ft, b = 6 ft



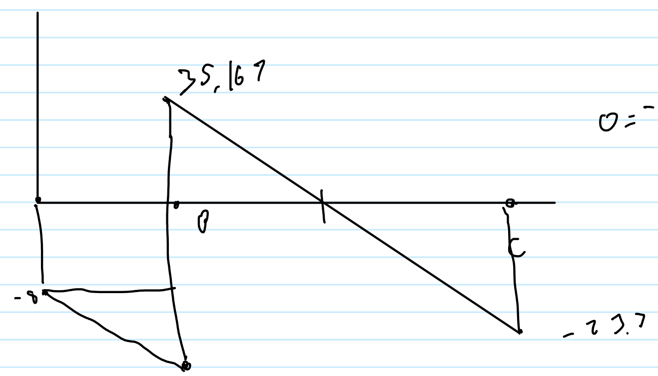
Determine the reaction force at A

$\Sigma M_B = -A_y a + \frac{a}{2} w a - P_b = 0 \Rightarrow A_y = 2950.7$ lbs

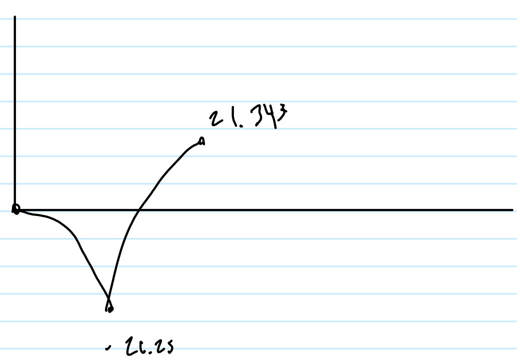
$\Sigma F_y = 495(14) - 2950.7 + B_y = 0 \qquad B_y = 3979.3$

$\Sigma M_B = C_y L_{BC} + P L_{AB} - wL(L_{BC} - L/2)$

$0 = -\frac{35.167 + 23.7}{4.5} x + 35.167$



$-8 - 13(1.5) = -27.5$



Bending Stress

Wednesday, February 22, 2023 4:14 PM

Algorithm of determining maximum bending stress in beams

Assume the moment (M) is acting about the horizontal axis (x-axis in the figure)

Step 1: Analyze the structure and determine the moment diagram

As described in section 5-2: Equilibrium in beams.

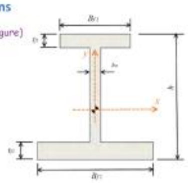
Step (2) Determine centroid and moment of inertia

As described in section 5-1: section properties.

centroid $\bar{x} = \frac{\Sigma x_i A_i}{\Sigma A_i}$ $\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i}$

Moment of inertia: $I = \Sigma(I_c + d^2 A)$

Note: It is recommended use tables shown here for calculation of centroid and moment of inertia



	A_i (mm²)	y_i (mm)	y_iA_i (mm³)
(1)			
(2)			

Table for determining centroid

	I_c (in.⁴)	$|d_i|$ (in.)	$d_i^2A_i$ (in.⁴)	I_i (in.⁴)
(1)				
(2)				
(3)				

Table for determining moment of inertia

Shape	Area (A)	Moment of Inertia (I)	Radius of Gyration (r)	Polar Moment of Inertia (J)
Semicircle	$A = \frac{\pi R^2}{2} = 1.571R^2$	$I_{x_0} = 0.1098R^4$, $I_{y_0} = I_x = \frac{\pi R^4}{8} = 0.3927R^4$	$r_{x_0} = 0.264R$, $r_{y_0} = r_x = \frac{R}{2}$	$J_{CG} = I_{x_0} + I_{y_0} = 0.5025R^4$, $J_o = \frac{\pi R^4}{4}$
Hollow Circle	$A = \frac{\pi(d^2 - d_1^2)}{4} = 0.7854(d^2 - d_1^2)$	$I_{x_0} = \frac{\pi(d^4 - d_1^4)}{64}$, $I_{y_0} = I_{x_0}$	$r_{x_0} = \frac{\sqrt{d^2 + d_1^2}}{4}$, $r_{y_0} = r_{x_0}$	$J_{CG} = \frac{\pi(d^4 - d_1^4)}{32}$
Hollow Rectangle	$A = bd - b_1d_1$	$I_{x_0} = \frac{bd^3 - b_1d_1^3}{12}$, $I_{y_0} = \frac{db^3 - d_1b_1^3}{12}$	$r_{x_0} = \sqrt{\frac{bd^3 - b_1d_1^3}{12A}}$, $r_{y_0} = \sqrt{\frac{db^3 - d_1b_1^3}{12A}}$	$J_{CG} = I_{x_0} + I_{y_0}$
Quarter-Circle	$A = \frac{\pi R^2}{4}$	$I_{x_0} = I_{y_0} = 0.0549R^4$, $I_x = \frac{\pi R^4}{16}$	$r_{x_0} = r_{y_0} = 0.2644R$, $r_x = 0.5R$	$J_{CG} = 0.1098R^4$

Algorithm of determining maximum bending stress in beams

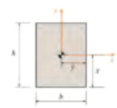
Assume the moment (M) is acting about the horizontal axis (z-axis in the figure)

Step (3) Determine bending stress

For symmetric sections: $\sigma^+_{max} = \sigma^-_{max} = \frac{Mc}{I}$

M: Internal moment in the beam.

c: Distance of the farest point of section from the horizontal axis

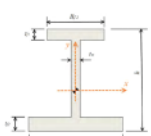


For non-symmetric sections:

Section subjected to positive moment: $\sigma^-_p = \frac{M^+_{max} c_{top}}{I}$ $\sigma^+_p = \frac{M^+_{max} c_{bot}}{I}$

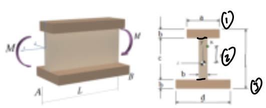
Section subjected to negative moment: $\sigma^+_n = \frac{M^-_{max} c_{top}}{I}$ $\sigma^-_n = \frac{M^-_{max} c_{bot}}{I}$

Maximum positive and maximum negative bending stress $\begin{cases} \sigma^+_{max} = \max(\sigma^+_p, \sigma^+_n) \\ \sigma^-_{max} = \max(\sigma^-_p, \sigma^-_n) \end{cases}$



Note: Positive bending stress causes tension on bottom of section and compression on top of it. Negative bending stress causes compression on bottom of section and tension on top of it.

A moment, M = 8.5 kip-ft is applied on two ends of the beam as shown in the figure. Point K is located e = 3.42 in above the centroid.
[a = 6 in, b = 2 in, c = 9 in, d = 11 in]



Centroid from bottom:

	A_i	y_i	A_iy_i
1	12	$2+9+\frac{2}{2}=12$	144
2	18	$2+\frac{9}{2} = 6.5$	117
3	22	$\frac{2}{2} = 1$	22
Σ	52		283

$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{283}{52} = \boxed{5.44 \text{ mm}}$

I:

	A_i	d_i	$A_i d_i^2$	$I_c = \frac{bh^3}{12}$
1	12	5.44 − 12 = −6.56	516.4	$\frac{6(2)^3}{12} = 4$
2	18	5.44 − 6.5 = −1.06	20.22	$\frac{2(9)^3}{12} = 121.5$
3	22	5.44 − 1 = 4.44	433.7	$\frac{11(2)^3}{12} = 7.3$

$516.4 + 4 + 20.22 + 121.5 + 433.7 + 7.3 = \boxed{1103 \text{ mm}^4}$

$\sigma_k = \frac{Mc}{I} = \frac{8.5(12)(3.42)}{1103} 10^3 = \boxed{316.2 \text{ psi}}$

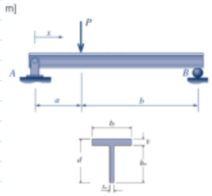
Section modulus for top:

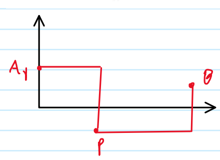
$\frac{I}{c_{top}} = \frac{1103}{2+2+9-5.44} = 146 \text{ in}^2$

Max tensile stress: $\frac{Mc_{top}}{I} = \frac{8.5(12)(2+2+9-5.44)(10^3)}{1103} = 698.8 \text{ psi}$

Max compression stress: $M_{[illegible]}$ $8.5(12)(5.44)\,10^3 = -503.2$ psi

For the cast iron beam shown, the maximum permissible compressive stress is $\sigma_{all,p}$ = 80 MPa and the maximum permissible tensile stress is $\sigma_{all,t}$ = 135 MPa. The section is WT265x37 which is a standard steel shape.[a = 2 m, b = 3 m]



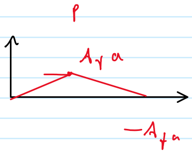


$\Sigma M_B = Pb - A_y(a+b)$

$A_y = \frac{Pb}{a+b} = .6P$

$\Sigma M_A = -Pa + B_y(a+b)$

$B_y = \frac{Pa}{a+b}$ $.4P$



$M_0 = -4500$

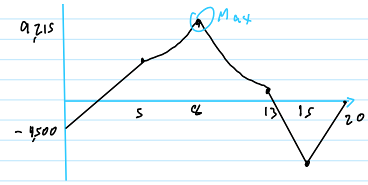
$M_5 = -4500 + 2110(5) = 6050$

$0 = -\frac{5600}{8}x + 2110 \Rightarrow x = 3.0$

$M_8 = 6050 + 2110(3) = 9215$

Max tensile stress: $\frac{Mc_{Top}}{I} = \frac{8.5(12)(2+2+9-5.44)(10^3)}{1103} = 698.8 \text{ psi}$

Max compressive stress: $\frac{Mc_{Bot.}}{I} = \frac{8.5(12)(5.44)}{1103} 10^3 = -503.2 \text{ psi}$



$0 = -\frac{5600}{8}x + 2110 \Rightarrow x = 3.0$

$M_8 = 6050 + \frac{2110(3)}{2} = 9215$

$M_{13} = 9215 - \frac{3490(8-3)}{2} = 490$

$M_{15} = 8240 - 3490(2) = -6490$

$M_{20} = -6490 + 1300(5) \approx 0$ ✓

$I_x = 1348 \text{ in}^4$

$\bar{y} = 5.8 \text{ in}$

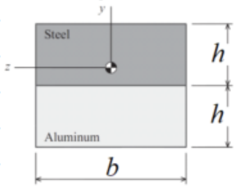
$\sigma^+ = \frac{9215(5.8)}{1348} = \boxed{39.65 \text{ psi}}$

$\sigma^- = \frac{9215(15-5.8)}{1348} = \boxed{62.89 \text{ psi}}$

Composite Beam Transformation

Monday, February 27, 2023 11:13 AM

An aluminum bar is bonded to a stell bar to form a composite beam as shown.[E_A= 7200 ksi, E_S= 31500 ksi, b = 2.6 in., h = 1 in.]



$\Rightarrow$

What is the distance to the centroid of the transformed aluminum section from the bottom surface of the beam?

$$b_t = \frac{E_S}{E_A} b = \frac{31500}{7200} 2.6 = 11.375$$

	A_i	y_i	Ay_i
T	11.375(1)	1.5	17.0625
B	2.6(1)	.5	1.3
Σ	13.975		18.3625

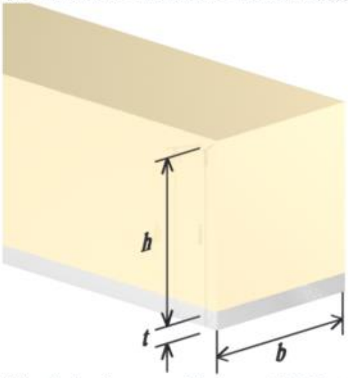
$$\frac{Ay}{A} = 1.314 \text{ in}$$

	A_i	d_i	$A_i d_i^2$	$I_c = \frac{bh^3}{12}$
T	11.375	1.314 − 1.5	.3935	$\frac{11.375(1)^3}{12} = .9479$
B	2.6	1.314 − .5	1.7227	$\frac{2.6(1)^3}{12} = .2167$

transform Σ = .3935 + .9479 + 1.7227 + .2167 = 3.2809
to Al

Transformed to steel $3.2809\left(\frac{7200}{31500}\right)$

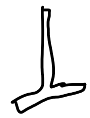
A composite beam is made of a wood block and a steel plate on its bottom side. Determine the bending stress on the top and the bottom part of the beam if the beam is subjected to a moment of M = 2.8 kN.m as shown in the figure.
[b=140 mm, h= 120 mm, t = 24 mm, E_{wood} = 10 GPa, E_{steel} = 210 GPa].



$b\left(\frac{10}{210}\right) = 6.6667$

H

⇒ allsteel

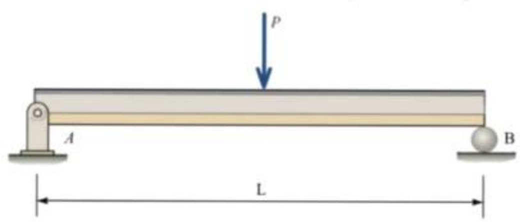


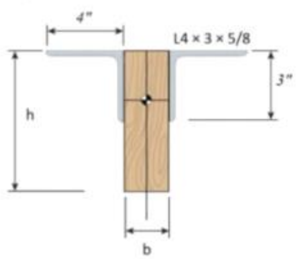
What is the distance of the centroid of the transformed section from the bottom of the section?

	A_i	y_i	$A_i y_i$
T	800	24+120/2	67200
B	3360	24/2	40320
Σ	4160		107520

$\bar{y} = 25.85$

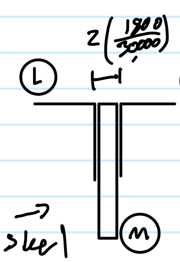
A simple beam that is L= 4 yd long supports a concentrated load of P. The beam is constructed of two angle sections, each L4 x 3 x 5/8 (4 in. side on horizontal face), on either side of a 12 x 2 in. wood beam. The modulus of elasticity of the wood and steel are E_{wood} = 1800 ksi and E_{steel}= 30000 ksi, respectively. Assume the allowable stresses in the steel and wood are 16 ksi and and 1.1 ksi, respectively. (Note: Disregard the weight of the beam)

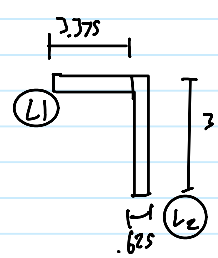




What is the allowable load P that can be applied on the beam?

	A_i	y_i	$A_i y_i$
L_1	2.109	12-.625/2	24.6533
L_2	1.875	12-3/2	19.6875
M	1.44	6	8.64
R_1	2.109	12-.625/2	24.6533
R_2	1.875	12-3/2	19.6875
Σ	8.739		97.32





$\bar{y} = \frac{97.32}{8.739} = 11.136$ in

A_i	$|\bar{y} - y_i|$	$A_i(\bar{y} - y_i)^2$	$\frac{bh^3}{12}$

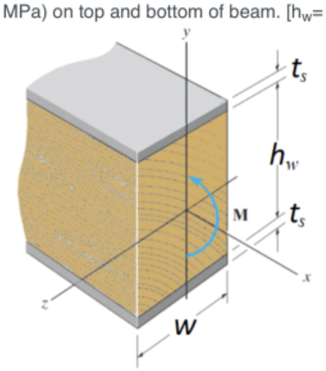
8.134

	A_i	$\lvert \bar{y} - y_i \rvert$	$A_i(\bar{y} - y_i)^2$	$\frac{bh^3}{12}$
L_1	2.109	.5515	.6414	.06866
L_2	1.875	.636	.75843	.59625
M	1.44	5.156	38.28	17.28
R_1	2.109	.5515	.6414	.06866
R_2	1875	.636	.75843	.59625
Σ			$I_x =$ 41.08	+ 18.61 = 59.69

$\sigma^+ = \frac{Mc}{I} \Rightarrow M = \frac{\sigma I}{c} \quad \frac{16 \quad (59.69)}{11.136} = \boxed{85.76 \text{ lbs}}$ Max

$\sigma^- = \frac{Mc}{I} \Rightarrow \quad M = \frac{\sigma I}{c} = \frac{1.1\,(59.69)}{12 - 11.136} = 75.994$ lbs

A wood beam (E= 16 GPa, $\sigma_{all,wood}$= 10 MPa) is reinforced with attaching steel plates (E= 210 GPa, $\sigma_{all,Steel}$= 180 MPa) on top and bottom of beam. [h_w= 170 mm, t_s= 9 mm, w = 95 mm]



$\Rightarrow$  Steel

$b = w \frac{16}{210} = 7.2381$

Determine the moment of inertia of the transformed section if the section is transformed to a full steel section.

	A_i	y_i	$A_i y_i$
T	9(95) = 855	$170 + 9 + \frac{9}{2}$	156892.5
M	7.2381(170) = 1230.5	9 + 170/2	115 664.762
B	9(95) = 855	$\frac{9}{2}$	3847.5
Σ	2940.47		276 404.8

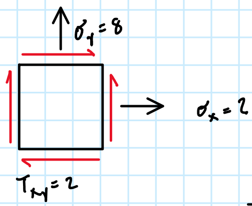
T: 4ω

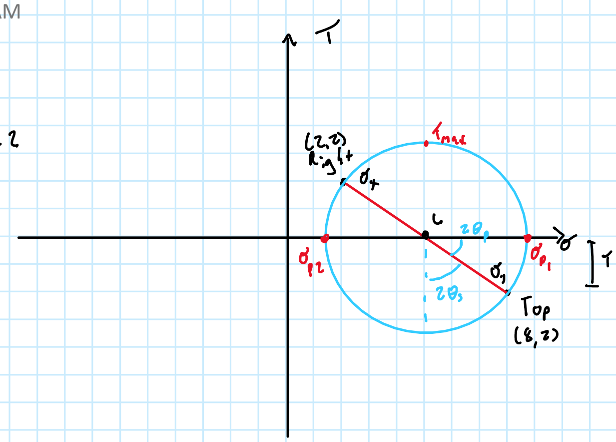
$$\sigma = \frac{Mc}{I}$$

$$\frac{\sigma I}{c}$$

Mohr's Circle

Monday, April 10, 2023 9:04 AM





$$\sigma_{p_1,p_2} = C \pm R$$

$$C = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\tau^2 + (\sigma_y - C)^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\theta_s + \theta_p = \frac{\pi}{4}$$

45